WS #15 - Kaplan-Meier CI

Math 150, Jo Hardin

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Your Name: ____

Names of people you worked with: ____

How many holes does a straw have?

Task: It can be shown that the natural log of the negative of the natural log of the Kaplan-Meier survival curve has a reasonably normal sampling distribution at each t.

$$ll\hat{S}(t) = \ln(-\ln \hat{S}(t)_{KM})$$

Consdier the SE of the natural log of the negative of the natural log of the Kaplan-Meier survival curve:

$$\begin{aligned}\hat{\sigma}^2(t) &= SE^2(ll\hat{S}(t)) \\ &= \frac{\sum_{i:t_i \leq t} \frac{d_i}{n_i(n_i - d_i)}}{\left[\sum_{i:t_i \leq t} \ln((n_i - d_i)/n_i)\right]^2}\end{aligned}$$

A 95% CI for $\ln(-\ln S(t))$ is:

$$ll\hat{S}(t)_{KM} \pm 1.96\hat{\sigma}(t)$$

Find a 95% CI for $S(t)$.

Solution:

$$
\begin{aligned}
ll\hat{S}(t) - 1.96\hat{\sigma}(t) &\leq llS(t) \leq ll\hat{S}(t) + 1.96\hat{\sigma}(t) \\
\exp(ll\hat{S}(t) - 1.96\hat{\sigma}(t)) &\leq -lS(t) \leq \exp(ll\hat{S}(t) + 1.96\hat{\sigma}(t)) \\
-l\hat{S}(t)\exp(-1.96\hat{\sigma}(t)) &\leq -lS(t) \leq -l\hat{S}(t)\exp(1.96\hat{\sigma}(t)) \\
l\hat{S}(t)\exp(-1.96\hat{\sigma}(t)) &\geq lS(t) \geq l\hat{S}(t)\exp(1.96\hat{\sigma}(t)) \\
l\hat{S}(t)\exp(1.96\hat{\sigma}(t)) &\leq lS(t) \leq l\hat{S}(t)\exp(-1.96\hat{\sigma}(t)) \\
\exp(l\hat{S}(t)\exp(1.96\hat{\sigma}(t))) &\leq S(t) \leq \exp(l\hat{S}(t)\exp(-1.96\hat{\sigma}(t))) \\
\exp(l\hat{S}(t))^{\exp(1.96\hat{\sigma}(t))} &\leq S(t) \leq \exp(l\hat{S}(t))^{\exp(-1.96\hat{\sigma}(t))} \\
(\hat{S}(t))^{\exp(1.96\hat{\sigma}(t))} &\leq S(t) \leq (\hat{S}(t))^{\exp(-1.96\hat{\sigma}(t))}
\end{aligned}
$$

A 95 % CI for $S(t)$ is

$$
\left(\hat{S}(t)_{KM}^{\exp(1.96\hat{\sigma}(t))}, \hat{S}(t)_{KM}^{\exp(-1.96\hat{\sigma}(t))}\right)
$$