WS #18 - Ionnidis - most published research is false

Math 150, Jo Hardin

Monday, April 14, 2025

Your Name: _____

Names of people you worked with: _____

What are you most excited about doing this summer?

Task: As defined in Ioannidis, "Why Most Published Research Findings are False"

R:

$$R = \frac{\#\text{ true relationships}}{\#\text{ null relationships}} \quad \text{in the population}$$

TRUE:

$$\begin{aligned} P(\text{study is true}) &= \frac{\#\text{ true relationships}}{\#\text{ total}} \\ &= \frac{\#\text{ true relationships}}{\#\text{ true} + \#\text{ null}} \\ &= \frac{\text{R}(\#\text{ null relationships})}{\text{R }(\#\text{ null}) + \#\text{ null}} \\ &= \frac{R}{R+1} \end{aligned}$$

size:

$$\begin{aligned} \alpha &= P(\text{type I error}) \\ &= P(\text{reject } H_0 | H_0 \text{ true}) \end{aligned}$$

power:

$$\begin{aligned} 1-\beta &= P(\text{reject } H_0 | H_0 \text{ false}) \\ \beta &= P(\text{type II error}) \\ &= P(\text{not reject } H_0 | H_0 \text{ false}) \end{aligned}$$

	True relationship		
Research Finding	Yes	No	Total
Yes	5.	4.	7.
No	3.	6.	8.
Total	1.	2.	c

tests:

$$c \quad = \quad \# \text{ of tests run}$$

Fill in the table (hint: I put numbers to suggest what I think is the easiest order for filling in the table). Find PPV (positive predictive value) = probability that the positive result is actually true.

Solution:

	True relationship		
Research Finding	Yes	No	Total
Yes	$c(1-\beta)R/(R+1)$	$c\alpha/(R+1)$	$c(R+\alpha-\beta R)/(R+1)$
No	$c\beta R/(R+1)$	$c(1-\alpha)/(R+1)$	$c(1-\alpha+\beta R)/(R+1)$
Total	$cR/(R+1)$	$c/(R+1)$	c

$$
\begin{aligned}
PPV &= \frac{c(1-\beta)R/(R+1)}{c(1-\beta)R/(R+1)+c\alpha/(R+1)} \\
&= \frac{c(1-\beta)R}{c(1-\beta)R+c\alpha} \\
&= \frac{(1-\beta)R}{(1-\beta)R+\alpha} \\
&= \frac{1}{1+\alpha/(1-\beta)R}
\end{aligned}
$$

$$
\begin{aligned}
PPV &> 0.5 \text{ more likely true} \\
\text{iff } (1-\beta)R &> (R-\beta R+\alpha)0.5 \\
(1-\beta)R0.5 &> \alpha 0.5 \\
(1-\beta)R &> \alpha
\end{aligned}
$$

$$
\begin{aligned}
PPV &< 0.5 \text{ more likely false} \\
\text{iff } (1-\beta)R &< \alpha
\end{aligned}
$$