WS #20 - Violating PH

Math 150, Jo Hardin

Friday, April 10, 2026

Your Name: ____

Names of people you worked with: ____

What are you most excited to do this summer?

Task: Consider the following hazard function / model on a single variable x which is binary (e.g., 0 is control, 1 is treatment).

$$h(t|x) = h_0(t)e^{\beta_1 x + \beta_2 x \ln(t)}$$

1. What is the HR (comparing $x = 1$ to $x = 0$)?
2. Let $\beta_1 = 0.5$ and $\beta_2 = -0.3$, compute the HR at $t = 1$, $t = e$, and $t = e^2$.
3. What is the effect of x over time?
4. At what time does that HR equal 1?
5. Sketch $HR(t)$ vs t. (Convince yourself that you see non-proportional hazards.)

Solution:

Note that the hazard function can be written as:

$$h(t \mid x) = h_0(t)e^{x(\beta_1+\beta_2 \ln(t))}$$

1. $HR(t) = e^{\beta_1+\beta_2 \ln(t)} = t^{\beta_2}e^{\beta_1}$
2. The HRs are:

- $t = 1 : HR = e^{0.5}$
- $t = e : HR = e^{0.5-0.3} = e^{0.2}$
- $t = e^2 : HR = e^{0.5-0.6} = e^{-.1}$

3. The effect of x changes over time! Early, x (e.g., the treatment) leads to a larger HR. But at larger times, the treatment leads to a smaller HR. (Can you see the interaction between x and time?) That is, the effect of x is dependent on the time.
4. Solve: $\beta_1 + \beta_2 \ln(t) = 0$. So, when $\ln(t) = -\beta_1/\beta_2$.
5.