WS #25 - Poisson coefficients

Math 150, Jo Hardin

Monday, April 27, 2026

Your Name: ____

Names of people you worked with: ____

What is the most interesting variable you've come across so far in the HELP study?

Task: Remeber #1 and #2? Just do #3.

1. Consider the linear regression model. Solve for β_1 as a function of $E[Y|x]$ and $E[Y|x+1]$. Write down the meaning of β_1 in words. (No cause!)

$$E[Y|x] = \beta_0 + \beta_1 \cdot x$$

2. Consider the logistic regression model. Solve for β_1 as a function of $E[Y|x] = p(x)$ and $E[Y|x+1] = p(x+1)$ (without the logit function). Write down the meaning of β_1 in words.

$$logit(p(x)) = \beta_0 + \beta_1 \cdot x$$

3. Consider the Poisson regression model. Solve for β_1 as a function of $E[Y|x]$ and $E[Y|x+1]$. Write down the meaning of β_1 in words.

$$E[Y|x] = e^{\beta_0 + \beta_1 \cdot x}$$

Solution:

1. To solve for β_1 we find the difference in expected values:

$$E[Y|x+1] - E[Y|x] = (\beta_0 + \beta_1 \cdot (x+1)) - (\beta_0 + \beta_1 \cdot (x)) = \beta_1$$

For each additional unit of x, the expected value of Y changes by β_1.

If $\beta_1 = 3$ we say, for each additional unit of x we predict the value of Y to be 3 units higher.

2. To solve for β_1 we find the difference in the logit functions:

$$logit(p(x+1)) - logit(p(x)) = (\beta_0 + \beta_1 \cdot (x+1)) - (\beta_0 + \beta_1 \cdot (x)) = \beta_1$$

However, we need β_1 as a function of $p(x)$ and $p(x+1)$. Recall that

$$logit(p(x)) = \ln\left(\frac{p(x)}{1-p(x)}\right)$$

$$\ln\left(\frac{p(x+1)}{1-p(x+1)}\right) - \ln\left(\frac{p(x)}{1-p(x)}\right) = \beta_1$$

$$\beta_1 = \ln\left(\frac{\frac{p(x+1)}{1-p(x+1)}}{\frac{p(x)}{1-p(x)}}\right)$$

That is, β_1 represents the natural log of the odds ratio for a one unit increase in x. e^{β_1} is the odds ratio for a one unit increase in x.

If $e^{\beta_1} = 3$ we say that for each additional unit of x, the odds of success are 3-fold higher.

3. e^{β_1} represents the ratio of means for a one unit increase in x.

$$\frac{E[Y|x+1]}{E[Y|x]} = \frac{e^{\beta_0+\beta_1\cdot(x+1)}}{e^{\beta_0+\beta_1\cdot x}} = e^{\beta_1}$$

If $e^{\beta_1} = 3$ we say that for each additional unit of x, the average Y is predicted to be 3 times bigger.