WS #6 - Maximum Likelihood Estimator

Math 150, Jo Hardin

Wednesday, February 12, 2025

Your Name: ____

Names of people you worked with: ____

What is your favorite dining hall at the 5Cs? How strongly do you feel about your choice?

Task:

Consider a toy example where you take a sample of size 4 from a binary population (e.g., flipping a coin that has probability heads of p) and get: failure, success, failure, failure (FSFF).

We want to find the value of p that *maximizes* the probability of your data. Try two approaches:

1. **Trial and error**: For 3 different values of p, find the probability of data FSFF. Which p gave you the highest probability?
2. **Calculus**: use calculus to find the value of p that maximizes the probability of data FSFF. (Hint, the ln of the probability is actually easier to maximize than the raw probability here.)

Solution:

1. Note that $P(FSFF|p) = p^1(1-p)^3$. Therefore:

$$\begin{aligned}
P(FSFF|p = 0.90) &= 0.0009 \\
P(FSFF|p = 0.75) &= 0.0117 \\
P(FSFF|p = 0.50) &= 0.0625 \\
P(FSFF|p = 0.47) &= 0.070 \\
P(FSFF|p = 0.25) &= 0.105 \\
P(FSFF|p = 0.15) &= 0.092 \\
P(FSFF|p = 0.05) &= 0.043
\end{aligned}$$

2. Let $L(p) = (P(FSFF|p))$

$$\ln L(p) = \ln(p^1(1-p)^3) = \ln(p) + 3 \cdot \ln(1-p).$$

$$\frac{\partial \ln L(p)}{\partial p} = \frac{1}{p} + \frac{3}{1-p} \cdot (-1) = 0$$

$$\frac{1}{p} = \frac{3}{1-p} \rightarrow 1 - p = 3p \rightarrow \hat{p} = \frac{1}{4}$$

(Note: the second derivative of $\ln L(p)$ is always negative, confirming that we've found a maximum, not a minimum.)