Math 150, Spring 2023
Jo Hardin
WU # 8
Thursday, 2/9/2023

Your Name: ____________________________________

Names of people you worked with: ____________________________________

1. Which door did you use to enter Estella (north, south, or west)?
2. Is an odds ratio defined as an odds which is calculated as success over failures? Or is an odds ratio defined as the ratio of two different odds (one for each of two groups)?
3. Consider a set up very similar to logistic regression. The response variable is still binary, and a generalized linear model is still applied. However, a different link function is used. The model is sometimes referred to as the log-linear model.[1]

$$\begin{aligned} E(Y|x) &= p(x) \\ \ln(p(x)) &= \beta_0 + \beta_1 x \end{aligned}$$

   In terms of β_0 and β_1, find:

   (a) The odds ratio for a one unit increase in x (your answer should depend on x).

   (b) The relative risk for a one unit increase in x (your answer should not depend on x).

Solution:

2. Odds ratios are defined by two separate calculations of odds – one for each of two groups.

$$p(x) = e^{\beta_0 + \beta_1 x}$$

3. (a)

$$\begin{aligned} odds(x) &= \frac{p(x)}{1-p(x)} = \frac{e^{\beta_0+\beta_1 x}}{1-e^{\beta_0+\beta_1 x}} \\ odds(x+1) &= \frac{p(x+1)}{1-p(x+1)} = \frac{e^{\beta_0+\beta_1 (x+1)}}{1-e^{\beta_0+\beta_1 (x+1)}} \\ OR &= \frac{odds(x+1)}{odds(x)} = \frac{\frac{e^{\beta_0+\beta_1 (x+1)}}{1-e^{\beta_0+\beta_1 (x+1)}}}{\frac{e^{\beta_0+\beta_1 x}}{1-e^{\beta_0+\beta_1 x}}} \\ &= e^{\beta_1} \cdot \frac{1-e^{\beta_0+\beta_1 x}}{1-e^{\beta_0+\beta_1 (x+1)}} \end{aligned}$$

3. (b)

$$RR = \frac{p(x+1)}{p(x)} = \frac{e^{\beta_0+\beta_1 (x+1)}}{e^{\beta_0+\beta_1 x}} = e^{\beta_1}$$

[1] n.b. As described in the notes, this model is often unstable when applied to a binary response variable because there is no way to keep $p(x)$ bounded below 1. The instability often leads to parameter estimates that do not converge / are not estimable via maximum likelihood. The inability of the model to converge is one of the main reasons we use logistic regression instead of log-linear models.